

# ON THE ASYMPTOTIC BEHAVIOR OF STOCHASTIC DIFFERENCE EQUATIONS

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## Abstract

In this paper we investigate the asymptotic behavior stochastic growth processes.

In this paper we investigate the asymptotic behavior of real-valued a recursively defined discrete time stochastic process  $X_n, n=1,2,\dots$  that satisfy stochastic difference equations of the form

$$X_{n+1} = X_n + g(X_n) + \xi_{n+1}, \quad (1)$$

where  $g(x)$  is positive function and  $(\xi_k, F_k)_{k \geq 1}$  is a square-integrable martingale difference sequence,  $F_0 \subset F_1 \subset \dots$  be an increasing sequence of  $\sigma$ -fields on some probability space. Let exists a measurable function  $\sigma^2$  such that

$$E(\xi_{n+1}^2 / F_n) = \sigma^2(X_n) \text{ a.e.}$$

and  $X_n \rightarrow +\infty$  a.e.

Stochastic difference equations of the form (1) arise in various stochastic models, see for examples, paper Keller G.F., Kersting G., Rösler U.(1987), Kersting G.(1992). Keller G.F., Kersting G., Rösler U.(1987) analyzed the equation (1) and give the necessary conditions for strong law large

number for  $G(X_n)$ , where  $G(x) = \int_1^x \frac{du}{g(u)}$  and  $\frac{X_n}{a_n} \rightarrow 1$  in probability, where  $a_n$  is a solution of

the deterministic equation  $a_{n+1} = a_n + g(a_n)$ , properly normalized  $X_n - a_n$  is asymptotically normal. Klebaner F.S.(1989) investigate case

$$g(x) = cx^\alpha + o(x^\alpha), c > 0, \alpha < 1, x \rightarrow +\infty$$

$$\sigma^2(x) = \sigma x^{1+\alpha} + o(x^{1+\alpha}), \sigma > 0, x \rightarrow +\infty$$

and proof that  $\frac{X_n^{1-\alpha}}{n}$  converge in distribution to the generalized gamma distribution. He applied the method of moments. Kersting G. (1992) generalized results Klebaner and showed that distribution of the variables  $n^{-1}G(X_n)$  weakly converge to gamma distribution. Anisimov V.V. (1989), (1991) apply the averaging principle and diffusion approximation method for investigation of recurrent processes the form(1).

Khusanbaev Ya.M., Rakhimov G.M. (1999) are interested in the asymptotic growth rates of the process

$$X_n(t) = n^{-\frac{1}{1-\alpha}} X_n, t \in \left[ \frac{k}{n}, \frac{k+1}{n} \right)$$

and show that under suitable conditions

$$\sup_{0 \leq t \leq T} \left| X_n(t) - [c(1-\alpha)t]^{\frac{1}{1-\alpha}} \right| \xrightarrow{P} 0$$

Khusanbaev Ya.M., Rakhimov G.M. (1997) study the diffusion approximation for stochastic process constructed by the sequence of variables

$$X_{nk+1} = X_{nk} + g_n(X_{nk}) + \xi_{nk+1}. \quad (2)$$

and we proof the theorems contained sufficient conditions for convergence's step processes defined on a sequence of random variables (2) to diffusion processes.

In this paper we give survey our result for processes (1),(2) and we give application method of characteristic function for convergence  $\frac{X_n^{1-\alpha}}{n}$  to gamma distribution.

## References

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